Search algorithm for a Common design of a Robotic End-Effector for a Set of 3D objects

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Abstract. We present an algorithm for defining the set of 3-finger contact point configuration which is suitable for grasping a given set of objects. The contact points configuration can be used for the design of a simple end effector capable of grasping this set of objects. Given a mesh of each object, the search algorithm maps all possible grasps for each object which satisfy a quality criterion and takes into account external loads applied to the object. The mapped grasps are represented by feature-vectors which represent the shape of the gripper in a 9-dimensional space and are registered in a database of all possible grasps for each object. Then we use a search algorithm for intersecting all points over all sets and find common feature vectors. Each common vector is the grasp configuration of an end-effector for grasping the group of objects.

Introduction

Robotic end-effectors in automation are designed for carrying out a specific action and for handling a specific part. The design phase of a typical end-effector consumes a considerable amount of engineering time and adds extra cost to the final product. Therefore, the purpose of this work is to develop an algorithm which will find a configuration of a simple end-effector for grasping a set of objects. Given a set of 3D CAD models of the parts, the algorithm will find a common configuration for grasping all of them. This configuration will serve for the design of a simple standard end-effector capable of grasping all objects in the given set.

This work uses the methods of force-closure [1],[2] and grasp quality measure [4] as criterion for determining and quantifying feasible grasps. To the best of our knowledge, no prior work has been done for searching common grasps of end-effectors for a set of objects. However, much work has been done in the area of 3D shape similarity comparison algorithms, such as [5]. This paper presents an algorithm for parameterization of the force-closure grasps of each object and using it for classification of the objects to possible grasps. In the first stage of the algorithm a *Force Closure Grasp Set* (*FCGS*) is constructed for every object by sampling all force-closure grasps, and representing the possible grasps as feature vectors in the high-dimensional space. Similarity join based on nearest-neighbor search is than done, for finding pairs of common feature vectors in the FCGS of all objects.

The following section is a background overview of grasping fundamentals used in this work. The generation of the FCGS and the similarity search for the common feature vectors are then presented. Finally, we present simulations of the proposed algorithm.

Background

Forces and torques can be represented as a *wrench* vector in the *wrench* space. A wrench is a 6-dimensional vector and is denoted as $\boldsymbol{w}=(\boldsymbol{f} \quad \boldsymbol{\tau})^T \in \mathbb{R}^6$ where $\boldsymbol{f} \in \mathbb{R}^3$ is a force vector and $\tau \in \mathbb{R}^3$ is a torque vector. Furthermore, a wrench originated from a force $\boldsymbol{f_i}$ applied at a contact point, c_i, can be described as $\boldsymbol{w_i}=(\boldsymbol{f_i} \quad \boldsymbol{p_i} \times \boldsymbol{f_i})^T$ where $\boldsymbol{p_i}$ is the location of contact point c_i represented in the object's coordinate frame.

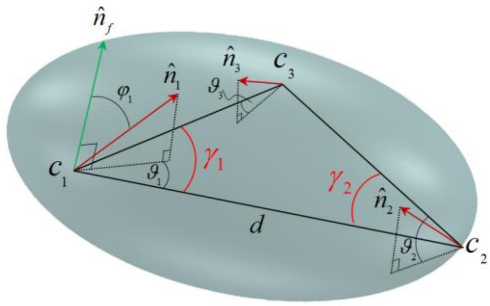


Fig. 1 - Three contact point grasp.

Friction exists at the contacts between the fingertips and the object's surface and can be represented by the Coulomb friction model. In this model, forces exerted in the contact point must lie within a cone centered about the surface normal. This is known as the *Friction Cone* (FC). The FC can be approximated by an *s*-sided convex polytope and the force exerted within the FC can be represented by a linear combination of the unit vectors $\hat{\boldsymbol{f}}_{ik}$ (primitive forces) constructing the polytope. The associated wrenches can be expressed by the primitive forces as

$$\boldsymbol{w_i} = \sum_{k=1}^{s} a_{ik}\boldsymbol{w_{ik}} = \sum_{k=1}^{s} a_{ik}\begin{pmatrix} \hat{\boldsymbol{f}}_{ik} \\ \boldsymbol{p_i} \times \hat{\boldsymbol{f}}_{ik} \end{pmatrix}, \tag{1}$$

where $\boldsymbol{w_{ik}}$ are the primitive wrenches associated with the primitive forces [3].

Force Closure. A grasp is said to be force-closure if it is possible to apply wrenches at the contacts such that any external forces and torques acting on the object can be counter-balanced. A system of wrenches can achieve force-closure when any external load can be balanced by a non-negative combination of the wrenches [1]. With system *W* of wrenches $\boldsymbol{w_1},\ldots,\boldsymbol{w_n}$, the convex-hull of *W* is defined as

$$CH(W) = \left\{ \sum_{i=1}^{n} a_i \boldsymbol{w_i} : \boldsymbol{w_i} \in W, \sum_{i=1}^{n} a_i = 1, a_i \geq 0 \right\} \tag{2}$$

The convex hull of the system of contact wrenches is denoted as the *Grasp Wrench Set* (GWS). A necessary and sufficient condition for a system of *n* wrenches $\boldsymbol{w_1},\ldots,\boldsymbol{w_n}$ to be force-closure is that the origin of $\mathbb{R}^k$ lies in the interior of the convex hull of the contact wrenches [1],[6]. Meaning

$$O \in \text{interior}\left(CH(W)\right) \tag{3}$$

Grasp quality measure. The grasp quality measure quantifies how much a grasp can resist an external wrench without fingers loosing contact or slip [7]. The grasp quality measure is defined as the distance from the origin of the GWS to the closest facet of *CH(W)*, and is given by

$$Q = \min_{w \in \partial W} \|w\| \tag{4}$$

where ∂W is the boundary of *CH(W)*[4].

FCGS generation algorithm

We generate the set of all possible grasps for each object. The grasps are represented as feature vectors in the *Force Closure Grasp Set (FCGS)*. This section presents the proposed structure of the grasp feature vector and the method for constructing the FCGS.

Grasp feature vector. A *3*-finger grasp of object *B* can be defined by a set of contact points, $P = (\boldsymbol{p_1}, \boldsymbol{p_2}, \boldsymbol{p_3})$ and the normals at each contact point $N = (\boldsymbol{n_1}, \boldsymbol{n_2}, \boldsymbol{n_3})$. This grasp definition can be mapped into a *d*-dimensional feature vector $\boldsymbol{e_j} = (u_1 \;\; \ldots \;\; u_d)^T$ injectively representing the grasp.

Fig. 1 shows a 3-finger grasp. The grasp can be represented as a triangle where the contact points are its vertices. The position of the 3 fingers relative to each other can be injectively represented as a triangle by two angles γ_1, γ_2 and the edge length between them *d*, given by the following equations:

$$\cos\gamma_1 = \frac{(p_3 - p_1)\bullet(p_2 - p_1)}{\|p_3 - p_1\|\|p_2 - p_1\|} \quad (5)$$

$$\cos\gamma_2 = \frac{(p_1 - p_2)\bullet(p_3 - p_2)}{\|p_1 - p_2\|\|p_3 - p_2\|} \quad (6)$$

$$d = \|p_1 - p_2\| \quad (7)$$

The normal to the surface at each contact point can be represented relative to the triangle by the angle φ_i between the normal of the plan of the triangle n_f, and the angle ϑ_i between the normal and each edge of the triangle. These angles are given by the following equations:

$$\varphi_i = \cos^{-1}\left(n_i \bullet n_f\right),\ i = 1,2,3\ , \quad (8)$$

$$\vartheta_i = \begin{cases} \frac{\pi}{2} - sgn\left(\left(\left(n_3 \times n_f\right) \times \hat{a}_{3,1}\right)\bullet n_f\right) \cdot \cos^{-1}\left(\left(n_3 \times n_f\right)\bullet \hat{a}_{3,1}\right), & i = 3 \\ \frac{\pi}{2} - sgn\left(\left(\left(n_i \times n_f\right) \times \hat{a}_{i,i+1}\right)\bullet n_f\right) \cdot \cos^{-1}\left(\left(n_i \times n_f\right)\bullet \hat{a}_{i,i+1}\right), & i = 1,2 \end{cases}, \quad (9)$$

where $\hat{a}_{i,j} = \frac{p_j - p_i}{\|p_j - p_i\|}$.

Therefore, the *3*-finger grasp can be injectively defined by a *9*-dimensional feature vector *e*,

$$e = (\gamma_1 \quad \gamma_2 \quad d \quad \varphi_1 \quad \varphi_2 \quad \varphi_3 \quad \vartheta_1 \quad \vartheta_2 \quad \vartheta_3)^T \quad (10)$$

It should be mentioned that there are three different representations for a triangle, depending on which edge is picked for defining *d*. Therefore, the longest edge of the triangle is always picked to represent *d*. This ensures that the algorithm will always treat the triangles from the same point of view.

Algorithm 1 FCGS generation

Input: Mesh of a 3D object *B* to be grasped and q - minimal grasp quality.
Output: FCGS $E = (\boldsymbol{e}_1, \ldots, \boldsymbol{e}_v)$ of object *B*.

1: Generate grasp *j* defined by $P_j = (\boldsymbol{p}_1, \ldots, \boldsymbol{p}_n)$.
2: **If** P_j is force closure and (grasp quality > q) **do**.
  i. Map grasp *j* to feature vector $\boldsymbol{e}_j = (u_1 \ \ldots \ u_d)^T$.
  ii. Label $\boldsymbol{e}_j$ as force closure.
  iii. Store link between $\boldsymbol{e}_j$ and P_j.
3: **Else** label $\boldsymbol{e}_j$ as non-force-closure and remove from *E*.
4: **If** the grasp space $E = (\boldsymbol{e}_1, \ldots, \boldsymbol{e}_v)$ is not fully labeled, **go to** step 1.
5: **Else Return** grasp space $E = (\boldsymbol{e}_1, \ldots, \boldsymbol{e}_v)$.

Grasp space generation. Generation of the FCGS is done by scanning the discretized mesh and going over all possible grasp feature vectors of object *B*, omitting those which are not force-closure or their quality is less than a given threshold. Algorithm 1 receives as an input a 3D mesh of a 3D CAD model. It samples all possible grasp feature vectors of an object and computes a set $E \in \mathbb{R}^d$

representing all combinations of *3*-points which achieve force-closure under the frictional contact constraint. This algorithm is based on the one originally proposed in [8].

Multi-sets common vectors search

Given *t* sets of vectors in a *9*-dimensional space $E_1,\ldots,E_t \in \mathbb{R}^d$ representing the FCGS of each object. A similarity algorithm takes the sets and output a set of vectors $Z \in \mathbb{R}^d$ which are common to two or more sets of $E_1,\ldots,E_t$.

Similarity Join. The main idea of the join algorithm is as follows: for each pair of sets E_i, E_j, we need to find vectors that are common to the two sets.

The join algorithm receives all FCGS data sets $E_1,\ldots,E_t \in \mathbb{R}^d$ and for each pair of sets, using nearest neighbor algorithm, match a vector in one set to the nearest vector in the other. After all the pairs are found, further inspection of whether they are close enough to be considered as the same grasp is done, thus, checking whether they are both within the predefined tolerances $\varepsilon_1,\ldots,\varepsilon_d$ along the major coordinates axes.

The mean vector of the pair found is checked if it exists in a registry set $Z \in \mathbb{R}^d$. Set *Z* is a cluster of vectors that are common to two or more sets of $E_1,\ldots,E_t$. Once a high-dimensional registry set *Z* of vectors $\boldsymbol{v_1},\ldots,\boldsymbol{v}_k \in Z$ is obtained, each vector $\boldsymbol{v}_i$ is coupled to the binary vector $\tilde{\boldsymbol{v}}_i \in \mathbb{R}^{1\times t}$ which denotes its existence in some of the primitive sets $E_1,\ldots,E_t$.

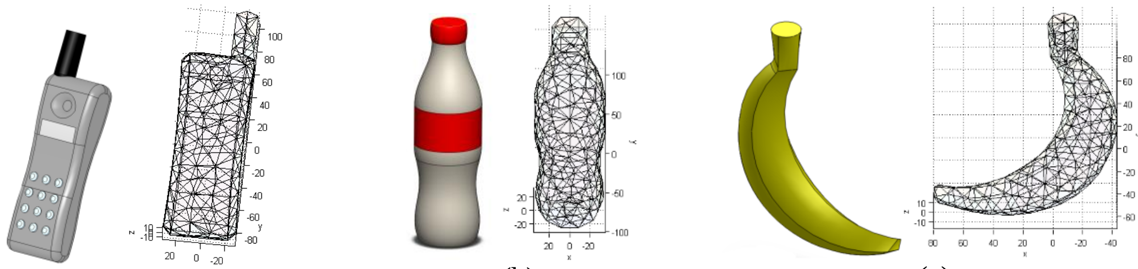
The next step is to find a vector within *Z* which exists in all of the primitive sets $E_1,\ldots,E_t$. Such a vector is said to cover the whole primitive sets $E_1,\ldots,E_t$. Let U_t be a *t*-dimensional vector set of size *v* consisting binary vectors, i.e., vector with *t* components consisting of 0's and 1's. For a vector $\boldsymbol{u}_i \in Z$, and its compatible vector $\tilde{\boldsymbol{u}}_i \in U_t$, it can be said that a vector $\boldsymbol{u}_i$ covers $E_1,\ldots,E_t$ if

$$\tilde{\boldsymbol{u}}_i = \left(\vec{1}\right)_{t\times 1} \tag{11}$$

Each vector found satisfying condition (11) represents a grasp configuration common to all objects.

Simulations

For simulations of the proposed method, the algorithm was implemented in MATLAB[1] on an Intel-Core i7-2620M 2.5GHz laptop computer with 8GB of RAM. The following simulations illustrate the working of the algorithm on 3-finger frictional grasps of three 3D objects.



(a) (b) (c)

Fig. 2 - Three objects to be grasped (first row) and their mesh (second row): (a) phone handset, (b) bottle and (c) banana.

The performance of the proposed algorithm is illustrated using the three 3D shapes shown in Fig. 2. The shapes are described with a mesh of *k* = 400 triangles representing their boundary. The friction coefficient was defined to be 0.6 and the friction cone was linearized using 5 triangles approximating the conical surface. Only grasps with quality measure greater than 0.1 are further considered.

[1] Matlab® is a registered trademark of Mathworks, Inc.

The length tolerances used in the similarity search $\varepsilon_1, \varepsilon_2, \varepsilon_3 [mm]$ were chosen in such way that the edges of the triangle will not extend or shorten more than 4% of their original length due to changes in the triangle‘s angles. Similarly, the angular tolerances $\varepsilon_4, \varepsilon_5, \varepsilon_6, \varepsilon_7, \varepsilon_8, \varepsilon_9, [^\circ]$ were defined such that each two query normals are within a cone whose angle is within 80% of the friction cone angle. The algorithm’s output are 16 solutions of common grasps for all objects with runtime time of 8 hours. Fig. 3 shows one of the 16 solutions with the highest quality measure (0.55).

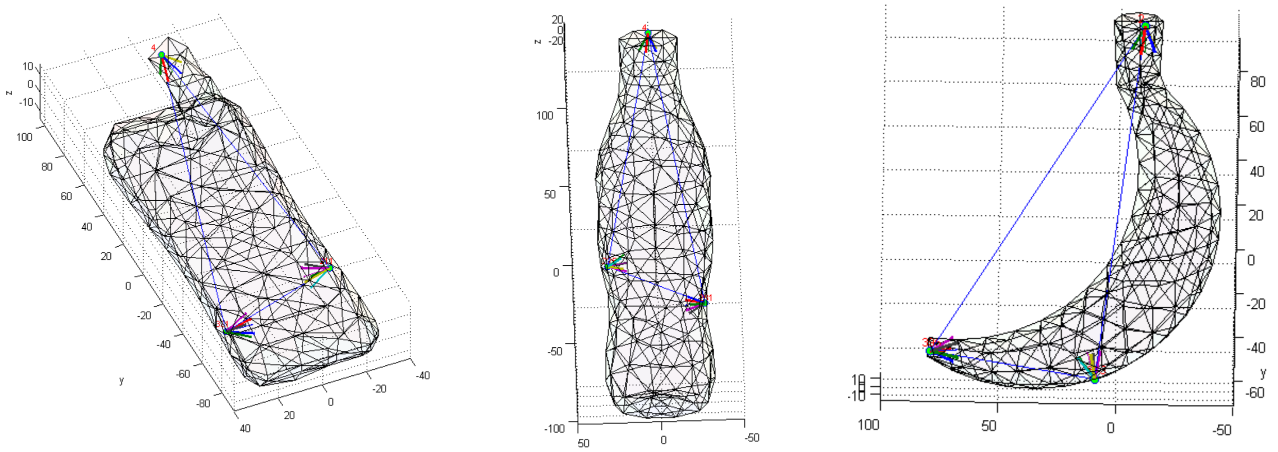


Fig. 3 - One grasp solution for all shapes.

Conclusions

An algorithm for finding a common grasp for a set of objects was proposed. The algorithm uses the notion of FCGS and high-dimensional similarity search. The feasibility of the algorithm was illustrated through simulations on 3D models. A set of grasp feature vectors was found that exist in all of the primitive sets. Each of these feature vectors is in fact a configuration of an end-effector used to grasp the object. Future work will involve improving runtime and validation experiments.

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